OPTIMIZATION OF THE AFT-BODY GEOMETRY OF AN AXI-SYMMETRIC SLENDER BODY TO MINIMIZE WAVE DRAG

By

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ABSTRACT

Extensive work on slender axi-symmetric bodies in transonic and supersonic flow has been carried out and well documented in the 1970's. With the progress of CFD that has taken place in the following decades, codes are available to look into the detailed flow characteristics and the resulting wave drag, in particular due to the geometry of the aft-body. However, to the authors' knowledge, the optimum drag of axi-symmetric bodies need further elaboration. Therefore it is the objective of present work look carefully into the drag mechanism due to geometry of the aft-body, in particular and more significantly, to find a way to optimize the total drag.

The objective of this work is to investigate an axi-symmetric slender body geometry which has a minimum drag, particularly in the transonic and supersonic flow regime. This research is started by conducting numerical computation studies of the previous existing axi-symmetric slender bodies to gain physical understanding of the effect of aft-body geometry to the overall wave drag. This CFD approach is conducted by using a commercially available flow solver called Fluents/Rampant. This code solves the basic equation of fluid flow for inviscid flow.

From this study, physical understanding of the phenomena, such as correlation between wave drag and wave distribution at the aft-body geometry, can be gained. The result of the study shows, for the body with pointed end, appearance of expansion wave at the aft-body and followed by a shock wave somewhere on the aft body. On the other hand, for the body with base area, an expansion fan appears at the corner connecting the aft body to the base. The expansion fan deflects the flow behind the body toward the centerline and flows comming from the upper and lower surface meet. The collision of the two streams results in the generation of shock waves somewhere downstream the base area. Based on the physical understanding gained above, a practical method of reducing the wave drag of a given body is developed for both cases, body with pointed end and body with base area. In this method, a shock wave generator is placed at a particular location on the aft body. By trial and error, a particular location of the shock wave generator placement is found which result in lowering the wave drag of the original body

Finally, another effort to minimize the wave drag of an axiymmetric cylindrical body is conducted by utilizing an optimization routine. A Modified Feasible Direction (MFD) based optimization program is developed. In this method, objective function uses the simple drag equation formulated by Von Karman and Moore. The MFD optimization program developed is intended to solve the two cases mentioned before; axi-symmtric slender body with pointed end and axi-symmetric slender body with base area. After validating the MFD optimization program by evaluating the existing axi-symmetric slender bodies, the program is used to search for optimum aft body geometries which minimize the wave drag. The results show that the MFD optimization program can be utilized in an aerodynamic optimization problem which has not been done before.

Key words : axi-symmetric slender body, optimization, Modified Feasible Direction, Computational Fluid Dynamic

1. Introduction

The present work started with practical interest of the author to develop an axisymmetric slender body geometry that has a minimum drag, particularly in the transonic and supersonic flow regime. Noting the transonic nature of the significant part of the prevailing flow field, a computational study has initially been performed on the aerodynamic characteristics of slender axi-symmetric bodies in transonic and supersonic flows.

This work will be carried out following three approaches; each is considered to have significant merit and contribution to the effort. The first approach will review selected analytical methods in the calculation of the aerodynamic characteristics of slender-axi-symmetric bodies in transonic flow; in particular the classical approach as described by Ashley and Landahl [4]. This method is analytical, elegant and reveals the generic contributions of geometrical elements to the drag, and will be useful in estimating the characteristics of several slender axisymmetric body configurations, such as those described by Krasnov [12], and Cuong & Norstrud [7], or other generic configurations. A more recent work (Biblarz and Priyono [5]) is of interest and introduced variable transformation that allows analytical solution of the linearized gas dynamic transonic equation. This method yields a family of slender axi-symmetric bodies as one of the closed form solutions of the equation. These aerodynamic bodies can be used as initial references in an optimization approach to obtain ones with certain prescribed aerodynamic characteristics, which is the main objective of the study.

The second approach will deal with the systematic determination of an optimum slender axi-symmetric body configuration based on the aerodynamic characteristics criteria. The simplest one is the minimization of the pressure (wave) drag. To this end, the minimization problem and procedure following (Vanderplaats[19]) will be formulated and optimization algorithms and codes will be developed in Fortran Code, in reaching the desired solution. In these two procedures, first order analytical approaches have been utilized, in order to have systematic, efficient and physically founded solution of candidate configurations.

The last approach will be devoted to look carefully into the more realistic flow fields of the candidate geometries using computational aerodynamic approach (and the use of commercially available CFD codes).

To this end, the numerical approaches are carried out to study slender bodies with various aft-body configurations and to arrive at an optimum drag configuration. This can be carried out by inspection or by incorporating the numerical evaluation in the optimization procedure, such as by following the philosophy described by Vanderplaats[22]. In the study, a numerical approach is carried out following the well established computational fluid dynamic method in solving the Euler equation around a slender axi-symmetric body in transonic and supersonic flow. The computational results have been validated by comparing the results for known geometric configurations and flow conditions with standard ones. The characteristics of the flow field can be utilized in working for the desired optimum characteristics, in particular obtaining geometry with minimum drag.

2. Statement of Work

The objective of this work is to find an optimum geometry of the aft-body of an axi-symmetric slender body with minimum wave drag at the transonic and supersonic flow. The kind of geometry considered in this work can be categorized into two groups, there are geometry with pointed ends and geometry with base area. To achieve the objective of this work, the following steps will be carried out.

1). To carry out computational studies of the well known axi-symmetric slender bodies to gain physical understanding of the effect of aft-body geometry to the overall wave drag.

2). To develop a general optimization program based on the Modified Feasible Direction (MFD) scheme. Then, the optimization program is used to obtain the aft-body geometries which have minimum wave drag.

3). To carry out computational study to validate the optimization results and to explain physically why such geometries have minimum wave drag.

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3. Result and Discussion

3.1 CFD Approaches of Existing Geometries

Firstly, the investigation using Analytical and CFD approach is addressed to the existing well-known geometries (MBB, Sears-Haack, Von Karman and Haack-Adams geometry). The goal of this investigation is to gain physical understanding of the effect of aft-body geometry to the overall wave drag of the existing axi-symmetric slender body with pointed ends as well as with base area.



Figure 1 Static Pressure Contour of Sears-Haack and MBB geometry At Mach number 1.2



Figure 2 Static Pressure Contour of Von Karman and Haack-Adams Geometry At Mach number 1.2

3.2 Geometry with Shock Generator

The aft-body wave distribution will be effected, by the changes of the aft-body shock wave, which is investigated further to obtain the relationship between the aft-body wave distribution and the wave drag. To this end, a shock generator (relatively small flat obstacle) will be placed to force a shock-wave to occur at a specific location. The shock generator is placed at various location at the rear part of the body, and the drag coefficient for the corresponding shock location will be recorded.



Figure 3 Sears-Haack Geometry With Shock Generator At Mach number 1.2



Figure 4 Body with Base area with Shock Generator At Mach number 1.2

Mach number	Dody Section	SEARS-HAACK GEOMETRY			
	bouy Section	ORIGINAL	WITH SHOCK-GEN		
1.2	fore	1.1367374	1.1312128		
	aft	-0.8046783	-0.80700031		
	total	0.3320591	0.32421248		
3.0	fore	0.29089237	0.29254983		
	aft	-0.073313161	-0.093617118		
	total	0.21757921	0.19893271		

 Table 1
 Cd of Sears-Haack With and Without Shock Generator

Table2 Cd of Body with Base Area With and Without Shock Generator

Mach number		CASE STUDY GEOMETRY			
	Body Section	Body C	Body C (Shock gen)		
1.2	fore	1.174821	1 17777		
	mid	-0.47456175	-0.566394		
	aft	-0.28257773	-0.201356		
	total	0.41768155	0.410022		
3.0	fore	0.33793453	0.339562		
	mid	-0.04357512	-0.055317		
	aft	-0.022621777	-0.022713		
	total	0.27173764	0.261530		

3.3 MFD Optimization Approach of Geometry With Pointed End

The MFD optimization program developed in this work has been validated with the existing geometry. The MFD program is applied for finding the MBB geometry which has a minimum wave drag. The geometry of interest in this case study is described in the paragraph below.

The optimization problem of the objective function (wave drag) will be subjected to a sets of constraints. The objective function of this investigation is defined as :

$$S(L) C_{D1} = -\frac{1}{\pi} \int_{0}^{L} \int_{0}^{x} S''(x) S''(\xi) \log |x - \xi| d\xi dx$$

The value of S''(x) can be determined by the geometrical properties of the candidate optimum geometry. An example is depicted in Figure 5.

The geometry is axi-symmetric slender body with pointed end, which has been normalized with the maximum length dimension of 1. The case study geometry consists of two parts namely fore body (Curve O-A) and aft-body (Curve B-C), connected by a cylindrical (A-B). This can be seen in the following figure.



Figure 5 Case Study Geometry with Pointed Ends

Result of MFD optimization program.

----- initial condition ----x(1) : 0.1000 initial objective function and gradient : f: -0.0346 df(1): -0.0232 _____ iter f x1 g(1) g(2) 0 -0.034621 0.10000000 -0.89999998 0.40000001 1 -0.045098 0.34721312 -0.65278685 0.15278688 2 -0.052842 0.44164041 -0.55835962 0.05835959 3 -0.056324 0.47770858 -0.52229142 0.02229142 8 -0.058543 0.5000000 -0.5000000 0.0000000 20 -0.359248 0.84823292 -0.15176708 -0.34823292 21 -0.359248 0.84823292 -0.15176708 -0.34823292 ----- optimization results -----: max. iteration : 21 objective func. : 0.3592

3.4 MFD Optimization Approach of Geometry With Base Area

The developed MFD optimization program has been validated with Von Karman geometry. Then the MFD program is applied to obtain a simple case study geometry, to find a geometry that has a minimum wave drag. This geometry of the case study considered consists of two parts, fore body and aftbody. The fore- body is an arbitrary existing axi-symmetrical slender body, whereas the aft-will be the object of optimization. The aft-body geometry of interest is represented by a function of F(x) (in this work a third order polynomial is assumed as an example) and the MFD program will be used to find the coefficient of the polynomial which leads to a minimum drag. The objective function of this investigation is the wave drag of the axi-symmetric slender body with base area as defined in Equation below,

$$C_{D1} = \frac{\left[S'(L)\right]^2}{2\pi} \log\left(\frac{2}{\lambda\sqrt{(L)}}\right) + \frac{1}{\pi} \left[S'(x)\int_0^x S''(\xi)\log(x-\xi)\,d\xi\right]_{x=0}^L - \frac{1}{\pi}\int_0^L \int_0^x S''(x)\,S''(\xi)\log(x-\xi)\,d\xi\,dx$$

The value of S"(x) can be determined by the geometrical properties of the candidate optimum geometry, an example is depicted in Figure 6 In this case, the case study geometry is axi-symmetric slender body with base area, which has normalized dimension of maximum length = 1 and maximum radius = 0.1. The case study geometry consists of two parts namely fore body (Curve O-A) and an aft-body consisting of a cylindrical (A-B) and curved (Curve B-C) parts with the base radius in this case assumed to be 0.5 of the maximum radius, as depicted in figure 6.



Figure 6 Case Study Geometry with Base Area

$$\begin{array}{lll} f_1(x) &\approx & R(x) = & a_f \, . x^3 \, + b_f \, . x^2 \, + c_f \, . x \\ f_2(x) &\approx & R(x) \, = rmax \\ R(x) &= & a.(x - X(1))^3 + X(2).(x - X(1))^2 + rmax \end{array}$$

Output of MFD Optimization Program

----- initial condition x(1) : 0.2000 x(2) : 2.0000 initial objective function and gradient : 1.4543 df(1): -6.0115 df(2): 1.9536 f: initial constraints and gradient: q(1): $2.0000 \, dg/dx(1):$ dg/dx(2): 1.0000 0.0000 $0.3000 \, dg/dx(1):$ -1.0000 dq/dx(2): g(2): 0.0000 dg/dx(1): 1.0000 dg/dx(2): g(3): -0.8000 0.0000 iter f x1 x2 g(1) g(2) g(3) _____ 0 1.45425 0.20000 2.00000 2.00000 0.30000 -0.80000 1 0.19539 0.69443 1.50562 1.50562 -0.19443 -0.30557 2 0.09479 0.69443 0.57509 0.57509 -0.19443 -0.30557 3 0.06823 0.69443 0.21967 0.21967 -0.19443 -0.30557 4 0.05972 0.69443 0.08391 0.08391 -0.19443 -0.30557 5 0.05671 0.69443 0.03205 0.03205 -0.19443 -0.30557 6 0.05559 0.69443 0.01224 0.01224 -0.19443 -0.30557 7 0.05517 0.69443 0.00468 0.00468 -0.19443 -0.30557 8 0.05501 0.69443 0.00179 0.00179 -0.19443 -0.30557 9 0.05494 0.69443 0.00068 0.00068 -0.19443 -0.30557

10 0.05491 0.69443 0.00000 0.00000 -0.19443 -0.30557 0.02370 0.49999 -0.03699 -0.03699 0.00001 -0.50001 11 12 0.01280 0.49999 -0.42414 -0.42414 0.00001 -0.50001 13 0.01280 0.49999 -0.42414 -0.42414 0.00001 -0.50001 _____ ----- optimization results -----: max. iteration : 13 objective func. : 0.0128 final design variables : x (1) : 0.5000E+00 x (2) : -.4241E+00 final objective function and gradient : 0.0128 df(1): 0.0587 df(2): 0.0022 f: final constraints and gradient: q(1): -0.4241 dg/dx(1): 0.0000 dq/dx(2): 1.0000 $0.0000 \, dg/dx(1): -1.0000$ dg/dx(2): g(2): 0.0000 g(3): $-0.5000 \, dg/dx(1): 1.0000$ dg/dx(2): 0.0000 Coef drag base on ref area = 0.4068913

3.4 CFD Validation Optimum MFD Geometry With Pointed End

CFD computational study is carried out to validate of the optimum of MBB geometry, which is a family of axi-symmetrical slender bodies with pointed ends. The CFD validation is conducted in the transonic and supersonic flow. It is carried out to look into the flow characteristics and to investigate the significant elements that give contribution to the wave drag.

Based on the result of the MFD optimization approach, the optimum of the case study MBB geometry with pointed ends is the geometry with starting curve of aft-body at X(1) = 0.85. In this validation not only the optimum geometry will be inspected but also cases which have starting point of curve upstream and down stream the optimum geometry. These geometries are also represented by using the polynomial order three ($f_{3x} = a.x^3 + b.x^2 + c.x + b.x^2 + c.x$

d) with the particular value of the polynomial coefficient. For convenience, will be introduced three kinds of case study geometry as follows,

(1) Body A is case study geometry with starting point of curve at 0.75 and the aft body polynomial is $f_3x = -6.4 x^3 + 14.4x^2 - 10.8x + 2$. (upstream of optimum geometry).

(2) Body B is case study geometry with starting point of curve at 0.85 and the aft body polynomial is $f_3x = -29.63 x^3 + 75.56x^2 - 64.22x + 18.30$ (optimum geometry).

(3) Body C is case study geometry with starting point of curve at 0.95 and the aft body polynomial is $f_3x = -800x^3 + 2280x^2 - 2166x + 686$. (downstream of optimum geometry).



CFD results

Figure 7.a Static Pressure Contour of Body A at Mach number 1.2



Figure 7.b Static Pressure Contour of Body B at Mach number 1.2



Mach numb er	Body Section	CASE STUDY GEOMETRY					
		BODY A		BODY B		BODY C	
1.2	fore	1.0	119.5 %	1.0	100 %	1.0	123.9%
	mid	0		0		0	
	aft	0.518		0.595		0.5	
3.0	fore	1.0	106.7 %	1.0	100 %	1.0	101.0%
	mid	0		0		0	
	aft	0.092		0.178		0.14	

Figure 7.c Static Pressure Contour of Body C at Mach number 1.2 Table 3 Result of MFD Validation of Geometry With Pointed End

The optimum case study geometry will reduce the drag coefficient of 19.5 % and 6.7 % at Mach number 1.2 and 3.0 respectively, compare with the Body A. It also reduces the drag coefficient of 23.9 % and 1.0 % at Mach number 1.2 and 3.0 respectively, compare with the Body C.

3.5 CFD Validation Optimum MFD Geometry With Base Area

CFD computational study is carried out to validate the optimum geometry which is a family of axi-symmetrical slender bodies with base area. As before the CFD validation is conducted in the transonic and supersonic flow regimes. This study is carried out to look into the flow characteristics and to investigate the significant elements that give contribution to the wave drag.

Based on the results of the MFD optimization approach, the optimum of the case study geometry with base area is the geometry associated with the starting point of the curve of aft-body at $X_{SC} = 0.5$ and polynomial of aft-body R(x) =

 $0.4482x^3 - 0.4241x + 0.1$. In this validation not only the optimum geometry will be inspected but also the geometries associated with the starting point of the curve located upstream and downstream of the optimum condition are investigated. These geometries are also described by using the third order polynomial with the particular values of the polynomial coefficients to be determined. For convenience, the following labels will be used to identify the three geometries considered,

(1) Body A is case study geometry with aft-body consisting of cone start at point A(0.5,rmax) and C(1.0, 0.05) with the equation of R(x) = -0.1x + 0.15.

(2) Body B is case study geometry with the starting point of the curve at X_{SC} = 0.5 and the polynomial of aft-body is $R(x) = 0.4482x^3 - 0.4241x + 0.1$.

(3) Body C is case study geometry with starting point of curve at $X_{SC} = X(1) = 0.6994$ and the coefficient of polynomial b = X(2) = 0.0.

Static pressure contour of the axi-symmetric slender body with base area under study are shown in Figure 8-a-b-c.



Figure 8.a Static Pressure Contour of Body A at Mach number 1.2



Figure 8.b Static Pressure Contour of Body B at Mach number 1.2



Figure 8.c Static Pressure Contour of Body C at Mach number 1.2

Mach no.	Body Section	CASE STUDY GEOMETRY					
		Body A		Body B		Body C	
1.2	fore	1.0	101.7%	1.0	100 %	1.0	135.8%
	mid	0.547		0.548		0.404	
	aft	0.188		0.191		0240	
3.0	fore	1.0	100.4%	1.0	100 %	1.0	114.7%
	mid	0.228		0.225		0.1295	
	aft	0.078		0.077 5		0.067	

Table 4. Result of MFD Validation of Geometry With Base Area

4. Conclusions

In the first investigation, computational studies of the previous existing axisymmetric slender body is conducted to gain physical understanding of the effect of aft-body geometry to the overall wave drag. This CFD computations are made by using a commercially available flow solver called Fluents/Rampant Ref[3]. All of the CFD numerical approach is conducted for the condition of inviscid flow at Mach number 1.2 and 3.0, so only the pressure drag contributes to the total drag.

The CFD calculations have been conducted to the axi-symmetric slender body with pointed end as well as the axi-symmetric slender body with base area. For the axi-symmetric slender body with pointed end the results indicate that the weak expansion occurs on the shallow curve, while the strong expansion occurs on the steep curve. Usually, the strong expansion is followed by the appearance of strong shock waves on the aft body and weak expansion is followed by weak shock waves. The distribution of expansion and shock waves at the aft-body of the axi-symmetric slender body with pointed end alters the total drag. Modifying the contour of aft-body geometry will change the wave distribution and consequently alters the total drag. The optimum geometry is a body which has set of shock and expansion waves' distribution, in such a way that minimizes the wave drag.

For the axi-symmetric slender body with base, the results indicate that there are no shock waves generated on the aft body region on the axi-symmetric slender body with base area. Instead, expansion fans appear at the corners connecting the aft body to the base. The corner expansion fans turn the flow toward the centerline. As a result, a low pressure region occurs at the base area of the body. Also, the expansion fans deflect the flow coming from the upper and lower surfaces toward each other. Somewhere behind the base of the body, the two streams would meet and shock waves form near that location.

Based on the physical understanding gained above, a practical method of reducing the wave drag of a given body is developed for cases, body with pointed end and body with base area. In this method, a shock wave generator is placed at a particular location on the aft body. By trial and error, a particular location of the shock wave generator placement is found which result in lowering the wave drag. By CFD analysis, it is found by placing the generator at this location, the aft body wave distribution induces a pressure distribution which lowers the wave drag of the body. This analysis indicates that there exists a particular shock wave generator location which results in generating an optimum aft body wave distribution and, hence, minimizes the wave drag of the body.

Finally, the third investigation is conducted to find the optimum geometry, which has minimum wave drag by utilizing a Modified Feasible Direction (MFD) optimization program. In this study, the first task is to develop a general optimization program base on the MFD scheme. The MFD optimization codes, which had been developed, are verified by evaluating a set of constraint that is consistent with the existing Sears-Haack geometry as well as the Von Karman geometry.

Next, the optimization program is used to obtain the optimum aft-body of The geometry of interest is axi-symmetric geometry of several case studies. slender body with pointed end and axi-symmetric slender body with base area. Both geometries consist of two parts, which is called fore-body and aft-body. The fore-body is the arbitrary body and is adopted from the fore-body of the MBB experimental body, for the case of body with pointed end. While the aftbody is formed by the polynomial order three and the coefficient of the polynomial treated as design variables. The fore-body for the case of body with base area is the same as the configuration for body with pointed end, and is adopted from fore-body of the MBB experimental body. The aft-body, in this case, consists of polynomial order three and has base area with radius which the size is half of the maximum radius Using the specific set of constraints, the MFD program is used to find the optimum geometry which has minimum wave drag. In other words, in this approach the MFD program searches for an aft body geometry that generates an optimum wave distribution for minimizing the wave drag of the body. The results of this study show that the MFD optimization program provides a convenient way to solve an aerodynamic optimization problem which has not been done before.

5. Recommendations

MFD optimization approach uses a very simple objective function of wave drag equation which is adopted from Von Karman and Moore. For further study, it is recommended using more complex equation (e.g. Navier-Stokes equation) by using a CFD solver. In that case, the drag output from CFD can be used as the objective function for MFD optimization program. In this work, inviscid flow is used for investigation and the body is moving without spinning on its axi. The use of NS solver allows the investigation of a more complex phenomena including shock boundary interaction and the effect of spinning of the body.

The practical methods of reducing the wave drag by utilizing shock wave generator need to be studied more extensively. Issues such as the size and shape of the shock wave generator should be examined carefully. Also, the trial and error procedure to find the best location for the placement of the shock wave generator should be replaces by an optimization procedure, perhaps by using the MFD program developed.

The experimental investigations are also recommended to validate the optimum axi-symmetric slender body studied in this work to ensure that the body has minimum drag.

Riwayat Hidup

Penulis dilahirkan pada tanggal 2 Mei 1952 di Madiun Jawa Timur. Lulus SMA Negeri I Madiun tahun 1970 dan memperoleh gelar Sarjana Teknik Mesin Institut Teknologi Sepuluh November Surabaya tahun 1979. Pada tahun 1994 memperoleh gelar Master of Science in Aeronautical Engineering di Naval Post Graduate School Monterey California USA.

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Penulis menikah dengan Susharyuni pada tahun 1978 dan mempunyai tiga orang anak, dua orang anak laki-laki dan satu orang anak perempuan, yaitu

Doddy Friestya 27 tahun, Meirina Priharyuningtyas 22 tahun dan Ilham Triwicaksono 11 tahun.

Prestasi Keilmuan

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